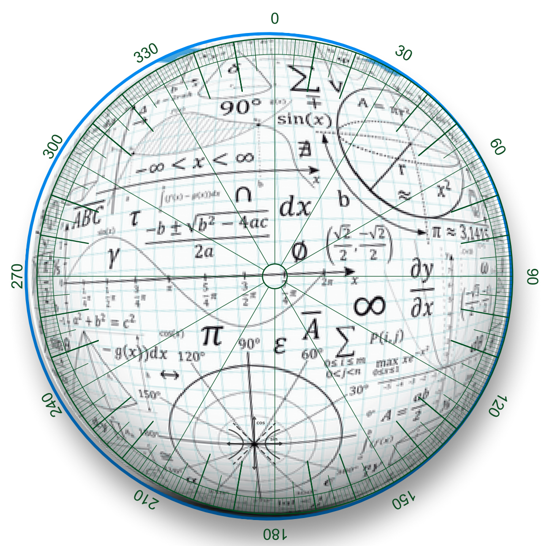
ZIAUDDIN UNIVERSITY
EXAMINATION BOARD

HSSC A Mathematics Syllabus



For exams in 2026 & onwards

INTRODUCTION TO ZUEB

The Ziauddin University Examination Board (ZUEB) is not only an awarding body but also a solution-driven educational organization dedicated to upholding the highest standards of academic excellence. ZUEB believes in excellence, integrity, and innovation in education. Established with a vision to foster a robust educational environment, ZUEB is committed to nurturing intellectual growth and development that meets international standards in an effective manner. The Ziauddin University Examination Board (ZUEB) was established through the Government Gazette No. XLI on June 6th, 2018. Its purpose is to ensure high quality, maintain global standards, and align the syllabi with national integrity within Pakistan's examination system. ZUEB manages student appeals, regulates assessments, and reviews policies to maintain high standards.

WHY CHOOSE HSSC-A AT ZUEB?

Ziauddin University Examination Board (ZUEB) offers the HSSC-A (Higher Secondary School Certificate Advance) program, designed for students from international educational backgrounds. This program provides a structured, affordable, and academically strong pathway for learners to align with Pakistan's education system. It allows students to fulfil national curriculum requirements, including Urdu, Islamiyat, Pakistan Studies, or Sindhi, with academic integrity and flexible learning options. ZUEB believes no student should be left behind due to financial limitations or cross-system transitions, and HSSC-A serves as a bridge between past efforts and future ambitions. It is the trusted choice for higher education in Pakistan

HSSC-ADVANCE MATHEMATICS

HSSC-Advance Mathematics at ZUEB is a foundation for exploring the language of logic and numbers, designed for students aspiring to pursue higher education in engineering, computer science, economics, actuarial sciences, and pure or applied mathematics. The course offers a rigorous, concept-driven curriculum aligned with both national and international standards, covering key topics such as algebra, calculus, geometry, trigonometry, probability, and statistics. Students develop a strong grasp of mathematical concepts and practical applications, while enhancing their problem-solving, logical reasoning, and analytical skills, ensuring they are both examination-ready and future-ready.

Aligned with national and international standards, HSSC-Advance Mathematics at ZUEB equips students with a comprehensive understanding of mathematical theories, methods, and applications that are essential for scientific and technological advancement. Designed for students aiming for careers in engineering, data science, finance, and academia, the course builds essential skills in quantitative analysis, abstract reasoning, and critical thinking.

Whether you are preparing for admission into top universities in engineering, science, or commerce, or planning a career in research, technology, or applied mathematics, HSSC-Advance Mathematics ensures you are academically prepared and nationally aligned, with a flexible, student-focused learning approach. Explore more on what HSSC-Advance Mathematics offers ZUEB HSSC-Advance Official Page.

Syllabus Overview

<table>
<tr><th>No.</th><th>Content</th><th>XII</th><th>XIII</th><th>AO</th><th>Exam Details</th></tr>
<tr><td>1</td><td>Pure Mathematics 1</td><td>✓</td><td>✓</td><td>AO1, AO2 and AO3</td><td rowspan="6">A combination of written exam papers (externally set and marked)

XII

Paper 1
Structured questions covering Pure Mathematics (1 & 2)
Duration: 2 hours
Weightage: 66%

Paper 2
Structured questions covering Mechanics 1
Duration: 1 hour 15 minutes
Weightage: 34%
<hr>XIII

Paper 1
Structured questions covering Pure Mathematics (1 & 2)
Duration: 2 hours
Weightage: 33.33%

Paper 3
Structured questions covering Pure Mathematics (1,2 & 3)
Duration: 2 hours
Weightage: 33.33%

Paper 4
Structured questions covering Mechanics (1 & 2) alongwith Probability and Statistics
Duration: 2 hours
Weightage: 33.33%</td></tr>
<tr><td>2</td><td>Pure Mathematics 2</td><td>✓</td><td>✓</td><td>AO1, AO2 and AO3</td></tr>
<tr><td>3</td><td>Pure Mathematics 3</td><td>-</td><td>✓</td><td>AO1, AO2 and AO3</td></tr>
<tr><td>4</td><td>Mechanics 1</td><td>✓</td><td>✓</td><td>AO1, AO2 and AO3</td></tr>
<tr><td>5</td><td>Mechanics 2</td><td>-</td><td>✓</td><td>AO1, AO2 and AO3</td></tr>
<tr><td>6</td><td>Probability and Statistics</td><td>-</td><td>✓</td><td>AO1, AO2 and AO3</td></tr>
</table>

Description of Assessment Objectives

AO1 – Use and Apply Standard Techniques

- Select and accurately perform routine procedures
- Recall facts, terminology, and definitions with precision

AO2 – Reason, Interpret, and Communicate Mathematically

- Develop rigorous mathematical arguments, including proofs
- Make logical deductions and inferences
- Judge the validity of mathematical reasoning
- Clearly explain processes and reasoning
- Use appropriate mathematical language and notation correctly

AO3 – Problem Solving in Mathematics and Other Contexts

- Translate real-world and abstract problems into mathematical processes
- Interpret solutions in their original context, assessing accuracy and limitation
- Formulate mathematical models for contextual situations
- Apply and work with mathematical models
- Evaluate modelling outcomes, recognize model limitations, and, where needed, suggest refinements

Weighting of Assessment Objectives

Assessment Objectives	P1 (%)	P2 (%)	P3 (%)	P4 (%)
AO1	45	45	40	30
AO2	35	35	35	40
AO3	20	20	25	30

Pure Mathematics 1

Aim: To develop an understanding of mathematical notation and core concepts in algebra, functions, trigonometry, and calculus, and will apply these foundational skills to solve problems and interpret advanced mathematical relationships.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
1.1	Understanding algebraic expressions	1.1.1	**Simplify** and **manipulate** algebraic expressions by **expanding** brackets, **collecting** like terms, **factorising**, and rearranging equations to extract required information (e.g., solving for roots of polynomials).	AO2
		1.1.2	**Interpret** and **apply** the notation of positive and negative rational indices.	AO1
		1.1.3	Apply index laws to simplify expressions: $a^m \times a^n = a^{m+m}$ $a^m \div a^n = a^{m-m}$ $(a^m)^n = a^{mn}$ $(ab)^n = a^n b^n$	AO2
		1.1.4	**Recognise** surds as expressions containing non-rational roots.	AO1
		1.1.5	Simplify expressions using surd laws: $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[m]{b}$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	AO2
		1.1.6	Identify conjugate pairs such as $(a + b)$ and $(a - b)$, and use the identity $(a + b)(a - b) = a^2 - b^2$.	AO1
		1.1.7	Rationalise denominators using conjugates, e.g. $\frac{1}{\sqrt{p}} \times \frac{\sqrt{p}}{\sqrt{p}} = \frac{\sqrt{p}}{p}$ $\frac{1}{\sqrt{p+q}} \times \frac{\sqrt{p+q}}{\sqrt{p+q}} = \frac{\sqrt{p+q}}{p+q}$	AO2
		1.1.8	**Present** irrational numbers in simplest surd form rather than converting to decimals.	AO1

1.2	Understanding quadratic equations	1.2.1	**Recognise** the general form of a quadratic equation in one variable, identifying the role of each constant and the condition that the leading coefficient is non-zero.	AO1
		1.2.2	**Factorise** quadratic expressions and **determine** solutions directly from the factors.	AO2
		1.2.3	**Identify** the number of possible real roots of a quadratic equation, **distinguish** between distinct and repeated roots, and **classify** the solutions accordingly.	AO3
		1.2.4	**Complete** the square to transform a quadratic equation into standard square form $a(x + b)^2 + c$, and **extract** solutions from this representation as $x =- b \pm \sqrt{-\frac{c}{a}}$.	AO2
		1.2.5	**Relate** the completed-square form $a(x + b)^2 + c$ of a quadratic equation to its graphical representation by **locating** the turning point (vertex) as $(-b, c)$.	AO2
		1.2.6	**Apply** the quadratic formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equations, and **analyse** the discriminant $b^2 - 4ac$ to determine the number and nature of the real roots.	AO3
		1.2.7	**Solve** quadratic equations and inequalities using appropriate methods such as factorisation, completing the square, or the quadratic formula, and **justify** the choice of method where appropriate.	AO3
		1.2.8	**Recognise** equations that are quadratic in form but involve a function of the variable (e.g $x = t$, $x = sin(t)$etc), and **apply** quadratic-solving methods to determine their solutions.	AO2
		1.2.9	**Solve** simultaneous equations involving one quadratic and one linear equation through substitution, and **interpret** the solutions in context.	AO3
		1.2.10	**Sketch** the graphs of quadratic functions, **identify** their key features such as turning points and intercepts, and **interpret** the relationship between the algebraic form and the graph.	AO3
1.3	Understanding functions	1.3.1	**Use** the standard symbols and **classify** numbers into real, integer, natural, and rational sets.	AO2
		1.3.2	**Describe** a map as a transformation from one set to another and **illustrate** how inputs correspond to outputs.	AO2

		1.3.3	**Differentiate** between one-to-one, many-to-one, and one-to-many mappings, and **categorise** them as onto or not onto.	AO3
		1.3.4	**Identify** the domain and range of a given mapping and **relate** them to the transformation process.	AO2
		1.3.5	**Characterise** a function as a mapping that is one-to-one or many-to-one, and **recognise** how this differs from other mappings.	AO2
		1.3.6	**Analyse** the properties of a map, including domain and range, and **evaluate** whether it qualifies as a function.	AO3
		1.3.7	**Apply** function composition using $f\ o\ g(x)$ or $f(g(x))$, and **solve** problems by combining two or more functions.	AO2
		1.3.8	**Sketch** and **interpret** transformations of basic functions of the form $y = g(x)$: $y = g(x) + c$ — vertical shift up by c units. $y = g(x + c)$ — horizontal shift left by c units. $y = cg(x)$ — vertical dilation (scale factor c). $y = g(cx)$— horizontal dilation with scale factor $\frac{1}{c}$ $y =-$ $g(x)$ — reflection across the x-axis. $y = g(-x)$ — reflection across the y-axis.	AO3
		1.3.9	**Apply** the properties of inverse of an one-to-one function $g(x)$: $gg^{-1}(x) = g^{-1}g(x) = x$ The graph of $g^{-1}(x)$ is the reflection of $g(x)$ on the line $y = x$ Domain of $g(x)$ = Range of $g^{-1}(x)$ Range of $g(x)$ = Range of $g^{-1}(x)$	AO3
		1.3.10	**Determine** the inverse of a given function **state** its domain and range, and **sketch** the function together with its inverse and the line $y = x$.	AO3
1.4	Understanding coordinate geometry	1.4.1	**Identify** the slope-intercept form of the equation of a straight line as $y = mx + c$, where m is the slope and c is the y-intercept.	AO1
		1.4.2	**Recognise** and **use** other forms of the equation of the line: General form → $ax + by + c = 0$ Slope-point form → $(y - y_1) = m(x - x_1)$	AO2
		1.4.3	**Determine** whether three points are collinear by establishing whether they lie on the same straight line.	AO3

		1.4.4	**Construct** the equation of a straight line from two given points (x_1, y_1) and (x_2, y_2) by calculating the gradient as $m = \frac{y_2 - y_1}{x_2 - x_1}$ and intercept as $c = y_1 - mx_1$	AO2
		1.4.5	**Formulate** the equation of a straight line given a point (x_1, y_1) and a gradient m by using the slope-point form: $(y - y_1) = m(x - x_1)$	AO2
		1.4.6	**Determine** whether two straight lines are parallel, perpendicular, or neither, using their slopes.	AO3
		1.4.7	**Calculate** the distance between two points on a line, **determine** the midpoint between them, and **find** the bounded area under a straight-line graph between two vertical limits = and = .	AO2
		1.4.8	**Determine** the midpoint of a line segment between two given points.	AO2
		1.4.9	**Compute** the area enclosed by a straight line, the x-axis, and two vertical boundaries.	AO2
		1.4.10	**Find** the point of intersection between two straight lines by solving their simultaneous equations.	AO2
		1.4.11	**Construct** the equation of the perpendicular bisector of a line segment.	AO2
		1.4.12	**Recall** and **use** the standard equation of a circle $x^2 + y^2 + 2\alpha x + 2\beta y + \gamma = 0$ with its centre at (− ,−) and radius given by $\sqrt{(\alpha^2 + \beta^2 - \gamma)}$.	AO2
		1.4.13	**Recognise** alternate forms of the circle equation and **determine** the centre and radius from them.	AO2
		1.4.14	**Determine** the points of intersection between a circle and a straight line by solving their simultaneous equations.	AO3
		1.4.15	**Construct** the equation of a tangent to a circle at a specified point.	AO2
		1.4.16	**Find** the perpendicular bisector of a chord of a circle and **use** it to reason about circle properties.	AO3
		1.4.17	**Determine** the points of intersection of a quadratic curve and a straight line, and **deduce** the corresponding quadratic solutions.	AO3

1.5	Understanding angles and trigonometry	1.5.1	**Interpret** the definition of a radian as a unit of angular measure and **convert** angles between degrees and radians.	AO2
		1.5.2	**Recall** and **use** the exact degree–radian conversions for common angles such as $30°, 45°, 60°, 90°, 180°$, and $360°$.	AO2
		1.5.3	**Apply** the formulas for arc length ($s = r\theta$) and sector area ($A = \frac{1}{2}r^2\theta$) to **solve** problems involving portions of a circle.	AO2
		1.5.4	**Sketch** and **use** the graphs of sine, cosine, and tangent functions for angles expressed in either degrees or radians.	AO2
		1.5.5	**Sketch** and **interpret** graphs of simple transformations of the sine, cosine, and tangent functions.	AO3
		1.5.6	**Recall** the exact values of sine, cosine, and tangent for $30°, 45°, 60°$, and related angles, and **apply** them in solving problems.	AO2
		1.5.7	**Use** the inverse trigonometric functions as $sin^{-1}x$, $cos^{-1}x$ and $tan^{-1}x$ to determine unknown angles within their principal ranges, **extend** solutions using trigonometric properties, and **justify** the appropriate choice of solution	AO3
		1.5.8	**Apply** fundamental trigonometric identities, such as the Pythagorean identity ($sin^2\theta + cos^2\theta \equiv 1$ and the relationship between tangent, sine, and cosine ($\frac{sin\theta}{cos\theta} \equiv tan\theta$ to **simplify** expressions and **prove** further identities.	AO3
		1.5.9	**Determine** all solutions of trigonometric equations within a specified interval, using knowledge of periodicity and symmetry.	AO3
		1.5.10	**Solve** right-angled triangle problems by applying trigonometric ratios to calculate unknown sides or angles.	AO2
		1.5.11	**Apply** the sine rule ($\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$) and cosine rule ($a^2 = b^2 + c^2 - 2(bc)cosA$ to find missing sides or angles in non-right-angled triangles.	AO2
		1.5.12	**Calculate** the area of a triangle using standard geometric methods or the formula involving sine of an angle between two sides: $A = \frac{1}{2} \times base \times height = \frac{1}{2}absinC$).	AO2

		1.5.13	**Determine** the area of a circular segment by applying geometric reasoning and trigonometric methods.	AO3
1.6	Understanding differentiation	1.6.1	**Interpret** differentiation as the limiting process of finding the gradient of a chord as its length approaches zero.	AO1
		1.6.2	**Describe** the derivative of a function at a point as the gradient there, and **generalise** the derivative as the gradient function across all points.	AO1
		1.6.3	**Recognise** and **use** the standard notation for limits $\lim_{h \to 0}$in the context of differentiation.	AO2
		1.6.4	**Use** different notations for derivatives: $\frac{d}{dx}$ → derivative with respect to x $$\frac{d}{dx}f(x) = \frac{dy}{dx} = f'(x) = y'$$	AO2
		1.6.5	**Differentiate** power functions of the form x^n where $n \neq 0$.	AO2
		1.6.6	**Differentiate** functions with multiple terms by applying differentiation rules term by term: If $y = u + v$, then $y' = u' + v'$.	AO2
		1.6.7	**Apply** the rule for differentiating constant multiples of functions and polynomials. $$\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$$	AO2
		1.6.8	**Construct** the equations of tangents and normal on the function $f(x)$ at point $x = a$ by using the following equation: Equation of tangent → $y - f(a) = f'(a)(x - a)$ Equation of normal → $y - f(a) = -\frac{1}{f'(a)}(x - a)$	AO3
		1.6.9	**Interpret** the second derivative as the rate of change of the gradient function.	AO3
		1.6.10	**Use** different notations for the second derivative and **express** it equivalently as the derivative of the derivative: $\frac{d^2}{dx^2}$ → second derivative with respect to x $$\frac{d}{dx}f'(x) = \frac{d^2y}{d^2x} = f''(x) = y''$$	AO2

1.7	Understanding integration	1.7.1	**Recognise** integration as the inverse process of differentiation, **identify** the integrand, and **understand** the role of the constant of integration.	AO1
		1.7.2	**Apply** the concept of an indefinite integral of $f'(x)$ as the family of functions that differentiate to $f'(x)$, and **use** standard integral $\int f'(x)dx = f(x) + c$ notation correctly.	AO2
		1.7.3	**Integrate** power functions of the form x^n where $n \neq -1$.	AO2
		1.7.4	**Integrate** functions with multiple terms by applying the rule of term-by-term integration: $\int f'(x) + g'(x)\ dx = f(x) + g(x) + c$	AO2
		1.7.5	**Integrate** polynomial functions accurately using standard rules.	AO2
		1.7.6	**Determine** the constant of integration using given conditions, such as a point through which the integral curve passes.	AO3

Pure Mathematics 2

Aim: Pure Mathematics 2 aims to build on the knowledge developed in Pure Mathematics 1, extending into more advanced concepts. It deepens understanding of algebra, trigonometry, and calculus, while introducing new areas such as exponentials, logarithms, sequences and series, and numerical methods for solving equations. Learners are expected to retain and apply prior Pure Mathematics 1 content, as it remains integral to assessment in this unit.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
2.1	Understanding algebra	2.1.1	**Recall** and **interpret** the meaning of the modulus (absolute value) function $\lvert f(x)\rvert$, where the argument is given by $f(x)$.	AO1
		2.1.2	**Construct** and **sketch** graphs of modulus functions of the form ($y = \lvert ax + b\rvert$) for linear functions and $y = f(\lvert x\rvert)$ for non-linear functions.	AO2
		2.1.3	**Determine** the solutions of equations involving modulus functions.	AO2
		2.1.4	**Manipulate** and **apply** relationships: $\lvert p\rvert = \lvert q\rvert \Leftrightarrow p^2 = q^2$ $\lvert x - p\rvert < q \Leftrightarrow p - q < x < p + q$ to **solve** equations and inequalities.	AO3
		2.1.5	**Perform** long division of polynomials (up to degree 4) and factorisation, clearly identifying quotient and remainder.	AO2
		2.1.6	**State** and **interpret** the Factor Theorem for the polynomial $p(x)$ for the following cases: $p(q) = 0 \Leftrightarrow (x - q)$ is a factor of $p(x)$. $p(\frac{q}{r}) = 0 \Leftrightarrow (rx - q)$ is a factor of $p(x)$.	AO1
		2.1.7	**Apply** the Factor Theorem to factorise higher-order polynomials.	AO2
		2.1.8	**State** and **interpret** the Remainder Theorem for polynomials divided by linear factors: If $p(x)$ is divided by $(rx - q)$, then the remainder is $p(\frac{q}{r})$.	AO1

		2.1.9	**Solve** polynomial equations and rational equations involving quotients of polynomials.	AO3
2.2	Understanding logarithms and exponentials.	2.2.1	**Interpret** and **apply** the definitions of logarithmic and exponential functions, and **demonstrate** their connection as inverse functions.	AO2
		2.2.2	**Relate** the natural logarithm to the mathematical constant e and **use** this link in problem solving.	AO2
		2.2.3	**Apply** the laws of logarithms: Multiplication Law: $log_a p + log_a q = log_a pq$ Division Law: $log_a p - log_a q = log_a \frac{p}{q}$ Power Law: $log_a (p)^n = n\ log_a p$ And the special use cases: $$log_a \left(\frac{1}{p}\right) = -log_a p$$ $log_p p = 1$, when $p > 0$ and $p \neq 1$ $log_a 1 = 0$ when $a > 0$ and $a \neq 1$	AO3
		2.2.4	**Transform** logarithmic expressions by changing their base and **justify** the equivalence of different forms. $$log_a(x) = \frac{log_b(x)}{log_b(a)}$$ $log_a(b) = \frac{1}{log_b(a)}$	AO3
		2.2.5	**Sketch** graphs of exponential functions (e^{ks}) with positive and negative growth rates (k positive and negative respectively), and the natural logarithm function (logarithm with the base e).	AO2
		2.2.6	**Solve** equations and inequalities where the variable appears in the exponent using appropriate techniques.	AO3
		2.2.7	**Transform** exponential equations into linear form using logarithms, and **solve** by interpreting the gradient and intercept.	AO3

2.3	Understanding sequences and series	2.3.1	**Represent** a series of terms using sigma notation $\sum_{i=a}^{b} f_i(x)$ and **evaluate** simple summations.	AO2
		2.3.2	**Apply** Pascal's Triangle to expand binomial expressions of the form $(a + b)^n$.	AO2
		2.3.3	**Recall** and **use** the definition of factorial notation, including special cases such as $0! = 1$.	AO2
		2.3.4	**Compute** combinations using the formula for nCr: $$nCr = \frac{n!}{r!\,(n-r)!}$$ and interpret their use in counting problems.	AO3
		2.3.5	**Determine** the r-th entry in the n-th row of Pascal's Triangle using $\frac{(n-1)!}{(r-1)!(n-r)!}$.	AO2
		2.3.6	**Expand** expressions of the form $(a + b)^n$ for $n \leq 5$ using the Binomial Theorem: $(a + b)^n = \sum_{k=0}^{n} \frac{n!}{r!(n-r)!} a^k b^{n-k}$.	AO2
		2.3.7	**Approximate** large powers using truncated binomial expansions, and estimate values when $0 < m < 1$.	AO3
		2.3.8	**Identify** arithmetic and geometric sequences, and **calculate** the n-th term as well as the sum of the first n terms.	AO3
		2.3.9	**Determine** whether a geometric series is convergent and **calculate** its sum to infinity where applicable.	AO3
2.4	Understanding trigonometry	2.4.1	**Solve** trigonometric equations of the form $sin(n\theta) = a, cos(n\theta) = a$, and $tan(n\theta) = b$ within a specified interval and **know** the ranges of their outputs ($-1 \leq a \leq 1$ and $-\infty \leq b \leq \infty$).	AO2
		2.4.2	**Solve** trigonometric equations of the form $sin(\theta + \alpha) = a, cos(\theta + \alpha) = a$, and $tan(\theta + \alpha) = b$ within a given interval and **know** the ranges of their outputs ($-1 \leq a \leq 1$ and $-\infty \leq b \leq \infty$).	AO2
		2.4.3	**Recall** and **use** the definitions of reciprocal trigonometric functions ($sec, cosec, cot$) and **determine** the values for which they are undefined.	AO2

		2.4.4	**Sketch** and **interpret** the graphs of $sec, cosec,$ and cot in degrees and radians.	AO3
		2.4.5	**Apply** the addition formulae to **solve** trigonometric equations and **prove** identities. $$sin\ P + sin\ Q = 2\ sin(\frac{P+Q}{2})\ cos\ (\frac{P-Q}{2})$$ $$sin\ P - sin\ Q = 2\ cos\ (\frac{P+Q}{2})\ sin\ (\frac{P-Q}{2})$$ $$cos\ P + cos\ Q = 2\ cos\ (\frac{P+Q}{2})\ cos\ (\frac{P-Q}{2})$$ $$cos\ P - cos\ Q = 2\ sin\ (\frac{P+Q}{2})\ sin\ (\frac{P-Q}{2})$$	AO3
		2.4.6	**Use** compound-angle formulae to solve trigonometric equations and **prove** identities: $$sin\ (P \pm Q) = sin\ P\ cos\ Q \pm cos\ P\ sin\ Q$$ $$cos\ (P \pm Q) = cos\ P\ cos\ Q \mp cos\ P\ sin\ Q$$ $Tan\ (P \pm Q) = \frac{tan\ P \pm tan\ Q}{1 \mp tan\ P\ tan\ Q}$ where $(P \pm Q) \neq (n + \frac{1}{2})\pi$	AO3
		2.4.7	**Derive** and **use** the double-angle formulae from addition formulae.	AO2
		2.4.8	**Express** sums of sine and cosine functions in the form of a single trigonometric function using R and α: $$a\ sin\ x \pm b\ cos\ x = R\ sin\ (x \pm \alpha)$$ $$a\ cos\ x \pm b\ sin\ x = R\ cos\ (x \mp \alpha)$$ Where $R > 0$, $0 < \alpha < \frac{\pi}{2}$, $R\ cos\ \alpha = a$, $R\ sin\ \alpha = b$ and $R = \sqrt{a^2 + b^2}.$	AO3
		2.4.9	**Apply** reciprocal trigonometric identities to solve equations and **prove** identities: $$sec^2\Theta \equiv 1 + tan^2\Theta$$ $$cosec^2\Theta \equiv 1 + cot^2\Theta$$	AO3
2.5	Understanding differentiation	2.5.1	**Identify** that a function $f(x)$ is increasing over an interval $[p, q]$ if $f'(x) \geq 0$ for all values of x such that $p < x < q$. It is strictly increasing if $f(x) > 0$ for all x such that $p < x < q$.	AO2
		2.5.2	**Identify** that a function $f(x)$ is increasing over an interval $[p, q]$ if $f'(x) \leq 0$ for all values of x such that $p < x < q$. It is strictly increasing if $f(x) < 0$ for all x such that $p < x < q$.	AO2
		2.5.3	**Identify** that a function $f(x)$ is unchanging over an interval $[p, q]$ if $f'(x) = 0$ for all values of x such that $p < x < q$	AO1

		2.5.4	**Determine** whether a function is increasing, decreasing, or constant over a region, and calculate the specific values of x where these conditions hold.	AO2
		2.5.5	**Find** the stationary points of a function by solving where the gradient equals zero.	AO2
		2.5.6	**Classify** a stationary point as a local maximum, local minimum, or point of inflection using the second derivative test.	AO3
		2.5.7	**Classify** a stationary point $x = a$ when the second derivative is zero ($f''(a) = 0$), by testing the sign of the first derivative on either side of the point.	AO3
		2.5.8	**Recall** and use the derivatives of standard functions including exponential (e^x), logarithmic ($\ln x$), sine ($\sin x$), cosine ($\cos x$), and tangent ($\tan x$).	AO2
		1.5.9	**Compute** derivatives of sums, differences, and constant multiples of functions.	AO2
		2.5.10	**Sketch** the graphs of functions, clearly showing the position and nature of stationary points.	AO3
		2.5.11	**Sketch** graphs of derivative (gradient) functions, **highlighting** the position and nature of stationary points.	AO3
		2.5.12	**Solve** contextual problems involving rates of change, where the derivative represents $\frac{\Delta y}{\Delta x}$.	AO3
2.6	Understanding integration	2.6.1	**Recall** and **use** the integrals of standard functions such as exponential (e^x), reciprocal ($\frac{1}{x}$), sine ($\sin x$), cosine ($\cos x$), and secant squared ($sec^2 x$).	AO2
		2.6.2	**Compute** integrals involving sums, differences, and constant multiples of functions.	AO2
		2.6.3	**Interpret** the definite integral as the evaluation of an integral between two limits, and **apply** the notation to calculate results: $\int_a^b f'(x)dx = f(a) - f(b)$.	AO2
		2.6.4	**Evaluate** definite integrals of functions with explicit and implicit limits.	AO2

		2.6.5	**Relate** the definite integral of a function to the area between the curve, the x-axis, and given vertical boundaries $x = a$ and $x = b$.	AO2
		2.6.6	**Recognise** that areas under the x-axis are negative and **separate** them appropriately in calculations.	AO2
		2.6.7	**Determine** areas enclosed by a curve and an axis using integration methods.	AO2
		2.6.8	**Calculate** the area between two functions using integration and/or geometric reasoning.	AO3
		2.6.9	**Apply** the Trapezium Rule to approximate the area under a curve that cannot be integrated analytically: $\int_a^b y dx = \frac{1}{2}h(y_0 + 2(y_1 + y_2 + y_3 + ... + y_{n-1}) + y_n)$ Where the step size $h = \frac{b-a}{n}$ and $y_i = f(a + ih)$ and **judge** whether the estimate is greater or less than the true value.	AO3
2.7	Understanding umerical methods	2.7.1	**Estimate** the roots of a function using graphical methods.	AO2
		2.7.2	**Detect** a root of a continuous function by identifying a sign change over an interval.	AO3
		2.7.3	**Approximate** the solutions of a function using iteration.	AO2
		2.7.4	**Use** graphical methods to **select** a suitable initial value for which an iterative formula will converge.	AO3
		2.7.5	**Recognise** that for the same function, different iterative formulae may converge to different roots, and that some may diverge depending on the initial value.	AO3

Pure Mathematics 3

Aim: Pure Mathematics 3 aims to build upon the foundations of Pure Mathematics 2, extending into advanced algebraic techniques and deeper calculus applications. This unit introduces new areas such as vectors and complex numbers, while strengthening the understanding of mathematical proofs. Prior knowledge of Pure Mathematics 1 and 2 is assumed and assessed throughout, even though not explicitly listed here.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
3.1	Understanding Mathematical proofs	3.1.1	**Identify** the structure and **understand** the requirements of a valid mathematical proof, including the logical flow and necessary assumptions.	AO1
		3.1.2	**Prove** an identity by carrying out algebraic manipulation, and **apply** the correct symbol (≡) to denote equivalence ("is always equal to").	AO2
		3.1.3	**Construct** mathematical arguments using Proof by Deduction, demonstrating a logical sequence from assumptions to conclusion.	AO3
		3.1.4	**Apply** Proof by Exhaustion to verify mathematical statements by examining all possible cases (where fewer than five cases are involved).	AO3
		3.1.5	**Disprove** mathematical statements by presenting counter-examples that invalidate a general claim. (Proof by counter-example)	AO3
		3.1.6	**Use** alternative proof strategies (e.g., Proof by Contradiction) to establish the validity of mathematical results.	AO3
3.2	Understanding partial fractions	3.2.1	**Recognise** that improper algebraic fractions occur when the degree of the numerator is greater than or equal to the degree of the denominator.	AO1
		3.2.2	**Apply** algebraic long division to **rearrange** an improper algebraic fraction into a mixed fraction form:	AO2

			$\frac{f(x)}{g(x)} = q(x) + \frac{r}{g(x)}$ where $q(x)$ is the quotient and r is the remainder obtained from long division.	
		3.2.3	**Decompose** a proper fraction with linear factors in the denominator (up to three factors) into partial fractions with unknown constants: $$\frac{f(x)}{(x+a)(x+b)(x+c)} = \frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{x+c}$$	AO2
		3.2.4	**Decompose** a proper fraction with linear factors, including one repeated factor (up to two factors), into partial fractions with the correct structure: $$\frac{f(x)}{(x+a)(x+b)^2} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+b)^2}$$	AO2
		3.2.5	**Decompose** an improper fraction with linear factors in the denominator (up to two factors) into partial fractions, where the polynomial part matches the degree requirement. $$\frac{f(x)}{(x+b)(x+c)} = A(x) + \frac{B}{(x+b)} + \frac{C}{(x+c)}$$	AO2
3.3	Understanding differentiation	3.3.1	**Differentiate** products of functions using the Product Rule: $$\frac{d}{dx}(u+v) = v.u' + u.v'$$	AO2
		3.3.2	**Differentiate** functions of functions using the Chain Rule: $$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$	AO2
		3.3.3	**Apply** the reciprocal case of the Chain Rule: $$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$ noting its limitation that $\frac{dx}{dy}$ is not a fraction but simply the reciprocal of .$\frac{dy}{dx}$.	AO2
		3.3.4	**Differentiate** quotients of functions using the Quotient Rule. $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.u' - u.v'}{v^2}$$	AO2
		3.3.5	**Use** the derivatives of the trigonometric functions sec, cot, and cosec (their derivatives are given in the formula sheet).	AO1

		3.3.6	**Differentiate** the inverse trigonometric functions $sin^{-1}x$, $cos^{-1}x$, and $tan^{-1}x$.	AO2
		3.3.7	**Apply** parametric differentiation to find $\frac{dy}{dx}$ when $x = f(t)$, $y = g(t)$.	AO2
		3.3.8	**Apply** implicit differentiation to find $\frac{dy}{dx}$ from relations involving x and y.	AO2
3.4	Understanding integration	3.4.1	**Use** standard integrals and inverse derivatives to integrate functions.	AO2
		3.4.2	**Apply** the chain rule in reverse to integrate functions of the form $f(ax + b)$.	AO2
		3.4.3	**Apply** trigonometric identities to integrate functions.	AO2
		3.4.4	**Solve** integrals of the form $\int \frac{f'(x)}{f(x)} dx = ln\|f(x)\| + c$.	AO2
		3.4.5	**Solve** integrals of the form $\int f'(x)(f(x))^n dx = \frac{f(x)^{n+1}}{n+1} + c$.	AO2
		3.4.6	**Apply** integration by parts when instructed: $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx.$	AO2
		3.4.7	**Apply** integration by substitution when given a substitution.	AO2
		3.4.8	**Use** partial fractions to integrate rational functions (when instructed).	AO2
3.5	Understand vectors	3.5.1	**Define** scalar and vector quantities, distinguishing between magnitude and direction.	AO1
		3.5.2	**Recognise** and **use** standard notation for scalars and vectors.	AO1
		3.5.3	**Apply** the fact that equal vectors represent parallel and equal line segments.	AO2

		3.5.4	**Use** vector equivalences: $\underline{AB}=\underline{BA}$, $\underline{AB} + \underline{BC} = \underline{AC}$, $\underline{AB} + \underline{BA} = 0$	AO2
		3.5.5	**Apply** the Parallelogram Law for vector addition.	AO2
		3.5.6	**Represent** and **compute** vectors in column form under addition and scalar multiplication.	AO2
		3.5.7	**Represent** and **compute** vectors in unit vector form ($\hat{\imath}, \hat{\jmath}, \hat{k}$), including scalar multiplication and addition.	AO2
		3.5.8	**Calculate** the magnitude of a vector using $\lvert v \rvert$.	
		3.5.9	**Express** vectors in magnitude–direction form, finding angle with positive x-axis using $\Theta = tan^{-1}(\frac{b}{a})$.	AO2
		3.5.10	**Solve** geometric problems in 2D using vector methods.	AO2
		3.5.11	**Represent** position vectors from origin to a point, and **use** them in vector differences: **AB** = **OB** - **OA**	AO1
		3.5.12	**Determine** the position vector of a point dividing a line segment AB in the ratio $AP: PB$ by using: **OP** = **OA** + $\frac{a}{a+b}$(**OB** - **OA**)	AO2
		3.5.13	**Apply** vector uniqueness in 2D: if *p***a**+*q***b**=*r***a**+*s***b**, then *p*=r, *q*=s.	AO2
		3.5.14	**Recognise** that vectors may have any number of dimensions.	AO1
		3.5.15	**Calculate** the magnitude 3D vector $v = (a, b, c)$ by using the formula $\sqrt{a^2 + b^2 + c^2}$ and find angles with coordinate axes using direction cosines: $cos(\theta_x) = \frac{a}{\lvert v \rvert}$ $cos(\theta_y) = \frac{b}{\lvert v \rvert}$ $cos(\theta_z) = \frac{c}{\lvert v \rvert}$	AO2

		3.5.16	**Calculate** the distance between two points $(x_1, x_2, x_3, ..., x_n)$ and $(y_1, y_2, y_3, ..., y_n)$ in n-dimensional space: $$d = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + ... + (y_n - x_n)^2}$$	AO2
		3.5.17	**Apply** uniqueness of vector representation in 3D: if p**a**+q**b**+r**c** = p'**a**+q'**b**+r'**c**, then coefficients are equal.	AO2
		3.5.18	**Formulate** and use the vector equation of a straight line in the form **r**=**a**+λ**b**.	AO2
		3.5.19	**Formulate** and use the vector equation of a line through two points **c** and **d**: **r**=**c**+λ(**d**−**c**).	AO2
		3.5.20	**Understand** and **apply** the scalar (dot) product: $a\ .\ b = \|a\|\|b\|cos\Theta$	AO2
		3.5.21	**Interpret** $a \cdot b$ as the magnitude of projection of one vector onto another.	AO2
		3.5.22	**Determine** perpendicularity of vectors using $a \cdot b = 0$.	AO2
		3.5.23	**Compute** the scalar product using components: $a\ .\ b = a_1b_1 + a_2b_2 + a_3b_3$	AO2
		3.5.24	**Find** the point of intersection of two non-parallel, non-skew lines in 3D.	AO2
		3.5.25	**Calculate** the angle between two intersecting lines using: $$cos\Theta = \left\|\frac{a.\ b}{\|a\|\|b\|}\right\|$$	AO2
3.6	Understanding complex numbers	3.6.1	**Recognise** that $\sqrt{-1}$ has no real solutions, and introduce the imaginary unit $i = \sqrt{-1}$.	AO1
		3.6.2	**Recognise** that imaginary numbers are of the form bi, and complex numbers of the form $a + bi$.	AO1
		3.6.3	**Identify** the real part Re(z) and the imaginary part Im(z) of a complex number $z = a + bi$.	AO1
		3.6.4	**Determine** equality of two complex numbers by comparing real and imaginary parts.	AO2

		3.6.5	**Perform** addition and subtraction of complex numbers.	AO2
		3.6.6	**Perform** multiplication of a complex number by a real scalar.	AO2
		3.6.7	**Perform** multiplication of two complex numbers in the form $a + bi$.	AO2
		3.6.8	**Use** the complex conjugate $z* = a - bi$ in calculations.	AO2
		3.6.9	**Show** and **use** that $z + z* = 2a$ is real.	AO2
		3.6.10	**Show** and **use** that $z . z* = a^2 + b^2$ is real.	AO2
		3.6.11	Perform division of complex numbers in the form $\frac{a+bi}{c+di}$.	AO2
		3.6.12	**Solve** quadratic equations with complex roots when $b^2 - 4ac < 0$.	AO2
		3.6.13	**Apply** the conjugate root theorem for polynomials with real coefficients.	AO2
		3.6.14	**Find** all real and complex roots of a cubic equation, given one complex root.	AO2
		3.6.15	**Find** all real and complex roots of a quartic equation, given one complex root.	AO2

Mechanics 1

Aim: Develop the ability to apply mathematical methods to model and analyse real-world physical situations. Build understanding of how forces influence the motion of objects, the conditions for equilibrium, and the principles of energy transfer in mechanical systems. This topic also aims to strengthen links between mathematical reasoning and physical interpretation.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
4.1	Understanding statics	4.1.1	**Recognise** that forces are vectors and use vector notation to represent them.	AO1
		4.1.2	**Construct** free-body diagrams for simple systems with multiple forces.	AO2
		4.1.3	**Resolve** forces into orthogonal components (not limited to horizontal and vertical).	AO2
		4.1.4	**Calculate** the magnitude and direction of the resultant of parallel and orthogonal forces.	AO2
		4.1.5	**Apply** the condition of equilibrium: resultant force is zero and vector sum of components in any direction is zero.	AO2
		4.1.6	**Determine** the force (magnitude and/or direction) required to maintain equilibrium of a particle.	AO2
		4.1.7	**Apply** Newton's Third Law to describe the equal and opposite nature of interacting forces.	AO1
		4.1.8	**Identify** Newton's Third Law pairs and represent them in force diagrams for a system.	AO2
		4.1.9	**Represent** contact forces using normal and frictional components.	AO1
		4.1.10	**Interpret** the concept of limiting equilibrium and the associated terminology.	AO3
		4.1.11	**Use** the coefficient of limiting friction (μ) in equilibrium situations, distinguishing between limiting and non-limiting cases.	AO2

		4.1.12	**Solve** equilibrium problems involving friction using $F = \mu R$ or $F \leq \mu R$ as appropriate.	AO2
4.2	Understanding Kinematics	4.2.1	**Recognise** displacement as a vector and distance as its scalar equivalent.	AO1
		4.2.2	**Recognise** velocity as the rate of change of displacement (vector) and speed as its scalar equivalent.	AO1
		4.2.3	**Recognise** acceleration as the rate of change of velocity.	AO1
		4.2.4	**Recognise** deceleration as the rate of decrease of velocity.	AO1
		4.2.5	**Solve** problems using: average velocity $=\frac{\Delta s}{\Delta t} = \frac{change\ in\ displacement}{time}$ and average acceleration $\frac{\Delta v}{\Delta t} = \frac{change\ in\ velocity}{time}$	AO2
		4.2.6	**Apply** SUVAT equations: $S = ut + \frac{1}{2}at^2$ $v = u + at$ $v^2 = u^2 + 2aS$ $S = \frac{1}{2}(u + v)t$ to solve constant acceleration problems.	AO2
		4.2.7	**Use** $g = 9.81\ m/s^2$ as the standard value of gravitational acceleration (and $g \approx 10\ m/s^2$ for estimation).	AO1
		4.2.8	**Sketch** displacement–time and velocity–time graphs.	AO2
		4.2.9	**Determine** instantaneous velocity from the gradient of a displacement–time graph.	AO2
		4.2.10	**Determine** total displacement from the area under a velocity–time graph.	AO2
		4.2.11	**Determine** instantaneous acceleration from the gradient of a velocity–time graph.	AO2
		4.2.12	**Apply** differentiation and integration with respect to time (Pure 1 knowledge) to model displacement, velocity, and acceleration.	AO2

		4.2.13	**Recognise** linear momentum as a vector given by p=mvp = mv.	AO1
		4.2.14	**Apply** the principle of conservation of linear momentum: total momentum before = total momentum after (if no external force acts).	AO2
		4.2.15	**Solve** problems involving collisions and explosions using conservation of momentum.	AO2
4.3	Understanding the action of resultant forces	4.3.1	**State** and **apply** Newton's First Law: "An object maintains its state of motion unless acted upon by an external resultant force."	AO1
		4.3.2	**State** and **apply** Newton's Second Law: an object accelerates when acted upon by an external resultant force.	AO1
		4.3.3	**Recognise** that Newton's Second Law implies force is proportional to the rate of change of momentum.	AO1
		4.3.4	**Solve** problems using $F = ma$ and $F = \frac{d(mv)}{dt}$.	AO2
		4.3.5	**Recognise** weight as the force due to gravity, and calculate it using $W = mg$.	AO1
		4.3.6	**Model** objects connected by light inextensible strings or light rigid bars as perfect connectors.	AO1
		4.3.7	**Model** smooth objects or surfaces as having no friction.	AO1
		4.3.8	**Solve** problems involving motion under constant forces, including motion on inclined planes.	AO2
		4.3.9	**Solve** problems involving non-constant forces by analysing a specific instant.	AO2
		4.3.10	**Solve** problems where two particles are connected by a perfect connector.	AO2
		4.3.11	**Recognise** impulse as the change in momentum,$\Delta p = mv - mu$.	AO1
		4.3.12	**Calculate** forces in collisions using $F = \frac{Impulse}{Time\ Taken}$.	AO2

		4.3.13	**Recognise** the coefficient of dynamic friction μ as the frictional ratio when an object is moving.	AO1
		4.3.14	Solve limiting equilibrium problems using $F = \mu R$.	AO2
4.4	Understanding energy, work and power	4.4.1	**Recognise** that work done is the energy transferred in a process, and that energy is the capacity to do work.	AO1
		4.4.2	**Recognise** that the work done by a force is proportional to the product of force and displacement in the direction of the force.	AO1
		4.4.3	**Solve** problems where displacement is not in the direction of the force using $W = Fdcos\quad\theta$.	AO2
		4.4.4	**Recognise** and use the kinetic energy formula:$KE = \frac{1}{2}mv^2$.	AO1
		4.4.5	**Recognise** and use the gravitational potential energy formula:$GPE = mgh$.	AO1
		4.4.6	**Recognise** and use the elastic potential energy formula:$EPE = \frac{1}{2}kx^2$.	AO1
		4.4.7	**Recognise** and use conservation of energy: total energy before = total energy after, if no external force acts.	AO1
		4.4.8	**Recognise** that the geometry of a particle's motion is irrelevant if energy is conserved, only initial and final configurations matter.	AO1
		4.4.9	**Solve** problems using the principle of conservation of energy.	AO2
		4.4.10	**Recognise** that power is the rate at which work is done.	AO1
		4.4.11	**Solve** problems using $P = \frac{W}{t} = Fv$.	AO2

Mechanics 2

Aim: To apply mathematical methods to study more complex types of motion, focusing on two-dimensional movement and objects that cannot be treated as particles.
Note: Knowledge from Pure Mathematics 1 is assumed and may be assessed within this unit, even though it is not listed here.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
5.1	Understanding projectile motion	5.1.1	**Resolve** a velocity vector into its horizontal and vertical components.	AO2
		5.1.2	**Identify** and **use** the fact that, without air resistance, horizontal acceleration is zero.	AO1
		5.1.3	**Identify** and **use** the fact that, without air resistance, vertical acceleration is equal to the acceleration due to gravity.	AO1
		5.1.4	**Interpret** projectile motion as independent vertical and horizontal components that can be analysed separately.	AO2
		5.1.5	**Apply** the SUVAT equations independently to vertical and horizontal motion.	AO2
		5.1.6	**Solve** problems involving projectile motion where the object returns to the same or a lower height than the launch point.	AO2
		5.1.7	**Determine** distances, speeds, and angles of motion at specific times or positions.	AO2
		5.1.8	**Calculate** the time of flight, i.e., the total duration between launch and return to the horizontal plane.	AO2
		5.1.9	**Calculate** the horizontal range, i.e., the distance travelled before the projectile returns to the horizontal plane.	AO2
		5.1.10	**Calculate** the maximum vertical height of a projectile when its vertical velocity becomes zero.	AO2

		5.1.11	**Describe** qualitatively how air resistance or varying gravitational acceleration affects projectile motion.	AO3
5.2	Understanding forces on extended objects. (moments and centre of mass)	5.2.1	**Interpret** a moment as the turning effect of a force, given by the force multiplied by its perpendicular distance from a pivot.	AO1
		5.2.2	**Calculate** moments using $M = Fd$ or $M = Fd\sin\theta$.	AO2
		5.2.3	**Identify** and use the fact that moments act in a clockwise or anticlockwise direction.	AO1
		5.2.4	**Calculate** a resultant moment by summing a set of moments.	AO2
		5.2.5	**Determine** the magnitude of a force or its distance from a pivot for equilibrium conditions (resultant force and resultant moment equal to zero).	AO2
		5.2.6	**Interpret** a couple as a pair of equal, anti-parallel forces.	AO1
		5.2.7	**Prove** that the resultant moment of a couple is equal to the magnitude of one of the forces multiplied by their separation, independent of the centre of rotation.	AO3
		5.2.8	**Apply** the fact that, with multiple pivots, moments can be taken about any one of them.	AO2
		5.2.9	**Apply** the fact that if a beam is about to rotate about one pivot, the reaction forces at all other pivots are zero.	AO2
		5.2.10	**Interpret** the centre of mass of an object as the point where its mass appears to act.	AO1
		5.2.11	**Apply** the fact that the centre of mass of a uniform bar is at its midpoint.	AO2
		5.2.12	**Use** moments to determine the centre of mass of a non-uniform bar.	AO2
		5.2.13	**Determine** the centre of mass of particles on a line using $\sum m_i x_i = \underline{x}\sum m_i$.	AO2
		5.2.14	**Prove** the relation $\sum m_i x_i = \underline{x}\sum m_i$ for particles on a line.	AO3

		5.2.15	Determine the centre of mass of particles in a plane using $\sum m_i x_i = \underline{x}\sum m_i$ and $\sum m_i y_i = \underline{y}\sum m_i$	AO2
		5.2.16	**Find** the centre of mass of simple uniform laminas (circle, rectangle, triangle).	AO2
		5.2.17	**Find** the centre of mass of a composite shape by combining or subtracting simple laminas.	AO2
		5.2.18	**Solve** equilibrium and dynamics problems involving non-uniform bars and uniform plane laminas.	AO2
5.3	Understanding circular motion and simple harmonic motion.	5.3.1	**Interpret** angular velocity ω as the angle turned per second (in radians) and use $\omega = \frac{\theta}{t}$.	AO1
		5.3.2	Apply the relationship between linear speed v and angular speed ω for circular motion of radius r, $v = \omega r$.	AO2
		5.3.3	**Interpret** frequency f as revolutions per second and period T as the time for one revolution.	AO1
		5.3.4	**Use** the relationship $f = \frac{1}{T} = \frac{\omega}{2\pi}$.	AO2
		5.3.5	**Demonstrate** that uniform circular motion is accelerated, and establish $a = \frac{V^2}{r} = r\omega^2$.	AO3
		5.3.6	**Use** Newton's laws to derive the centripetal force$F = \frac{mv^2}{r} = mr\omega^2$, directed to the centre.	AO3
		5.3.7	**Calculate** quantities such as linear speed, angular speed, centripetal acceleration, centripetal force, or radius of circular motion.	AO2
		5.3.8	**Show** that the x-coordinate of circular motion about the origin is $x = R\cos(\omega t)$.	AO3
		5.3.9	**Differentiate** to obtain velocity and acceleration in the x-direction: $v = -R\omega\sin(\omega t)$, $a = -R\omega^2\cos(\omega t)$.	AO3

		5.3.10	**Interpret** simple harmonic motion (SHM) as motion under the restoring force $F =- kx$, where xx is displacement from equilibrium.	AO1
		5.3.11	**Demonstrate** that SHM follows the equations $x = Acos \quad (\omega t)$, $v =- A\omega sin \quad (\omega t)$, $a =- A\omega 2cos \quad (\omega t)$, and link to circular motion with $\omega = \sqrt{\frac{k}{m}}$.	AO3

Probability and Statistics

Aim: To study complex systems by analysing large numbers of similar events through averaging. The aim is to evaluate models of random events and interpret data arising from experiments.
Note: Prior knowledge of Pure Mathematics 1 is assumed and may be assessed within this unit, though it is not restated here.

	The learner will:	SLO #	Assessment Criteria - The learner can:	Cognitive levels
6.1	Understanding probability	6.1.1	**Demonstrate** awareness that probability measures the likelihood of an event and is expressed between 0 (impossible) and 1 (certain).	AO1
		6.1.2	**Demonstrate** awareness of the terms experiment, event, outcome, sample space, mutually exclusive, independent, equally likely, and biased in the context of probability.	AO1
		6.1.3	**Apply** the fact that the sum of probabilities of all possible events equals one.	AO2
		6.1.4	**Use** $P(event)$ notation to represent probabilities.	AO1
		6.1.5	**Calculate** probabilities of specific outcomes in simple dice and card experiments (single roll or draw).	AO2
		6.1.6	**Use** sample space diagrams, Venn diagrams, tree diagrams, and histograms to represent probability spaces and extract probabilities.	AO2
		6.1.7	**Demonstrate** awareness that the intersection of A and B means "A and B," denoted by $A \cap B$.	AO1
		6.1.8	**Demonstrate** awareness that the union of A and B means "A or B," denoted by $A \cup B$.	AO1
		6.1.9	**Demonstrate** awareness that the complement of A means "not A," denoted byA'.	AO1

		6.1.10	**Apply** the rule for mutually exclusive events: $P(A \cup B) = P(A) + P(B)$.	AO2
		6.1.11	**Apply** the rule for independent events: $P(A \cap B) = P(A) \times P(B)$, and use the multiplication rule to test independence.	AO2
		6.1.12	**Use** conditional probability notation $P(A\|B)$ to represent the probability of A given B and $P(A\|B')$ to represent the probability of A given 'not B'.	AO2
		6.1.13	**Apply** the fact that for independent events $P(A\|B) = P(A\|B') = P(A)$ and $P(B\|A) = P(B\|A'') = P(B)$, and **use** them to determine the independence of events.	AO2
		6.1.14	**Apply** and **justify** the addition formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, with reasoning using Venn diagrams or other methods.	AO3
		6.1.15	**Apply** and justify the multiplication formula $P(A \cap B) = P(A\|B) \times P(B)$, with reasoning using Venn diagrams or other methods.	AO3
		6.1.16	**Solve** probability problems involving conditional and/or successive events.	AO2
6.2	Understanding representation and characterisation of data	6.2.1	**Demonstrate** awareness of the difference between qualitative vs. quantitative data, and discrete vs. continuous data, and **choose** appropriate representations.	AO1
		6.2.2	**Construct** histograms, box plots, and stem-and-leaf diagrams for data sets, and **extract** information from them.	AO2
		6.2.3	**Use** grouped frequency tables to organise data into classes and **determine** class boundaries, midpoints, and widths.	AO2
		6.2.4	**Demonstrate** awareness that measures of location describe position in a data set, while measures of central tendency describe the centre.	AO1
		6.2.5	**Calculate** mean, median, and mode (or modal class) for a set of data.	AO2

		6.2.6	**Calculate** the combined mean of two data sets using the weighted mean formula: If set A has the size n_a and mean $\underline{x}_a$ and set B has n_b and mean $\underline{x}_b$, then measure the mean of A and B is: $$\underline{x} = \frac{n_a\,\underline{x}_a + n_b\,\underline{x}_b}{n_a + n_b}$$	AO2
		6.2.7	**Calculate** the mean from a frequency table using $\underline{x} = \frac{\sum xf}{\sum f}$.	AO2
		6.2.8	**Determine** quartiles and percentiles as measures of location for a set of data.	AO2
		6.2.9	**Apply** interpolation to estimate the median, quartiles, and percentiles from grouped frequency tables.	AO2
		6.2.10	**Determine** quartiles from grouped continuous data or cumulative frequency tables.	AO2
		6.2.11	**Demonstrate** awareness that measures of spread describe the distribution of data as a single value.	AO1
		6.2.12	**Calculate** range, interquartile range (IQR), and interpercentile range (IPR).	AO2
		6.2.13	**Calculate** the summary statistic: $S_{xx} = \sum(x - \underline{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$.	AO2
		6.2.14	**Calculate** variance using $\sigma^2 = \frac{S_{xx}}{n}$ as a measure of spread.	AO2
		6.2.15	**Calculate** standard deviation using $\sigma = \sqrt{\frac{S_{xx}}{n}}$	AO2
		6.2.16	**Apply** data coding of the form $y = \frac{(x-a)}{b}$.	AO2
		6.2.17	**Calculate** the mean and standard deviation of coded data, and convert back to the original mean $\underline{x} = b\underline{y} + a$ and standard deviation $\sigma_x = b\sigma_y$.	AO2
		6.2.18	**Identify** outliers using given definitions or impossibility arguments, and **clean** the data.	AO3

		6.2.19	**Identify** positive skew, negative skew, or symmetry in data from frequency tables, bar charts, or box plots (without calculation).	AO1
		6.2.20	**Compare** two data sets using measures of location, spread, and skewness.	AO3
6.3	Understanding discrete random variables	6.3.1	**Recognise** that a discrete random variable takes only specific numerical values, with outcomes unknown before the experiment (e.g. dice roll).	AO1
		6.3.2	**Recognise** that a probability distribution is uniform if each outcome is equally likely (e.g. fair dice).	AO1
		6.3.3	**Use** probability notation for random variables, $P(X = x)$.	AO1
		6.3.4	**Construct** tables or diagrams to represent the probability distribution of a discrete random variable.	AO2
		6.3.5	**Calculate** the cumulative distribution function $F(x) = P(X \leq x)$.	AO2
		6.3.6	**Construct** cumulative distribution tables for discrete random variables.	AO2
		6.3.7	**Calculate** the expected value of a discrete random variable, $E(X) = \sum xP(X = x)$.	AO2
		6.3.8	**Calculate** the variance of a discrete random variable,$Var(X) = E[(X - E(X))^2] = E(X^2) - (E(X))^2$.	AO2
		6.3.9	**Recognise** that if X is a discrete random variable, then $f(X)$ is also a discrete random variable.	AO1
		6.3.10	**Calculate** the expected value of a function, $E(f(X)) = \sum f(X)P(X = x)$.	AO2
		6.3.11	**Use** expectation properties: $E(aX + b) = aE(X) + b$ and $E(X + Y) = E(X) + E(Y)$.	AO1
		6.3.12	Use variance property: $Var(aX + b) = a^2Var(X)$.	AO1
		6.3.13	**Use** the results for a uniform distribution over $[1,2,3, \ldots, n]$:$E(X) = \frac{n+1}{2}$, $Var(X) = \frac{(n+1)(n-1)}{12}$.	AO2

6.4	Understanding the normal distribution	6.4.1	**Recognise** that a continuous random variable can take any value within a range, and that $P(X = x) = 0$.	AO1
		6.4.2	**Use** the fact that probability in a continuous distribution is given by the area under the curve, with total area = 1.	AO1
		6.4.3	**Recall** properties of the Normal Distribution: defined by μ, σ^2; symmetric; mean=median=mode; bell-shaped; inflection point at $\mu \pm \sigma$; empirical rule *(68% – 95% – 99.75%)*.	AO1
		6.4.4	**Use** the notation $X \sim N(\mu, \sigma^2)$ for a normally distributed variable.	AO1
		6.4.5	**Recall** that the Standard Normal Distribution is $Z \sim N(0, 1^2)$.	AO1
		6.4.6	**Apply** standardisation:$Z = \frac{X-\mu}{\sigma}$, and use it to find or infer unknown means/variances.	AO2
		6.4.7	Use diagrams or Normal tables to find probabilities, or determine values of X given $P(Z > z) = p$.	AO2
6.5	Understanding correlation and regression	6.5.1	**Identify** bivariate data as pairs of values of two variables.	AO1
		6.5.2	**Plot** bivariate data on scatter diagrams and interpret patterns.	AO2
		6.5.3	**Distinguish** between the independent (explanatory) variable on the x-axis and the dependent (response) variable on the y-axis.	AO1
		6.5.4	**Interpret** correlation as the linear relationship between two variables, recognising its direction and strength.	AO1
		6.5.5	**Express** linear regression as the regression line of yy on x, of the form $y = ax + b$.	AO1

		6.5.6	**Calculate** summary statistics: $S_{xx} = \sum(x - \underline{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$, $S_{yy} = \sum(y - \underline{y})^2 = \sum y^2 - \frac{(\sum y)^2}{n}$, $S_{xy} = \sum xy - \frac{\sum x \sum y}{n}$.	AO2
		6.5.7	**Calculate** the least squares regression line using $a = \frac{S_{xy}}{S_{xx}}$, $b = \underline{y} - a\underline{x}$, and apply it for interpolation.	AO2
		6.5.8	**Recognise** the product moment correlation coefficient (PMCC) as a measure of strength of linear association.	AO1
		6.5.9	**Calculate** PMCC using $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$.	AO2
		6.5.10	**Apply** coding to one or both variables and confirm that PMCC remains unchanged.	AO2
		6.5.11	**Draw** conclusions from regression and correlation, and **apply** extrapolation to estimate values outside the data range.	AO3

Formula Booklet

1. Algebra

a. Basic Algebraic Identities

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

b. Laws of indices

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^0 = 1\ (for\ a \neq 0)$$

c. Laws of Logarithms

$$log_a(xy) = log_a(x) + log_a(y)$$

$$log_a\left(\frac{x}{y}\right) = log_a(x) - log_a(y)$$

$$log_a(x^n) = n\ log_a(x)$$

$$log_a(b) = \frac{log_c(b)}{log_c(a)}$$

d. Quadratic Equations

Standard form: $ax^2 + bx + c = 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Completing the square: If $p(x+q)^2 + r = 0$, then $x = -q \pm \sqrt{\frac{-r}{p}}$

e. Arithmetic Progression & series

$$u_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(a+l) = \frac{n}{2}(a+u_n)$$

f. Geometric Progression & series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \; where \; (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \; where \; (|r| < 1)$$

g. Binomial Expansion

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

h. Summation formulae

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$$

i. Modulus properties

$$|a| = |b| \Leftrightarrow a^2 = b^2$$

$$|x-a| < b \Leftrightarrow a-b < x < a+b$$

2. Geometry

a. Circle properties

Arc Length: $s = r\theta$

Sector Area: $A = \frac{1}{2}r^2\theta$

b. Triangles

Area: $A = \frac{1}{2}\, ab\, sinC$

Sine rule: $\frac{a}{sin\, A} + \frac{b}{sin\, B} + \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Pythagorean Theorem: $a^2 = b^2 + c^2$ (in a right triangle)

c. 3D solids

Surface area of a sphere: $A = 4\pi r^2$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Surface area of a cone: $A = \pi rl + \pi r^2$

Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

Volume of a pyramid: $V = \frac{1}{3} \times base\ area \times h$

Surface area of a cylinder: $A = 2\pi rh + 2\pi r^2$

Volume of a cylinder: $V = \pi r^2 h$

3. Trigonometric Identities

a. Basic Definitions

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

b. Pythagorean Identities

$$\sin^2\theta + \cos^2\theta \equiv 1$$
$$\sec^2\theta \equiv 1 + \tan^2\theta$$
$$\csc^2\theta \equiv 1 + \cot^2\theta$$

c. Compound angle identities

$$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \; where \; (A \pm B) \neq \left(n + \frac{1}{2}\right)\pi$$

d. Double angle identities

$$\sin(2A) \equiv 2\sin A\cos A$$

$$\cos(2A) \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan(2A) \equiv \frac{2\tan A}{1 - \tan^2 A} \; where \; (2A) \neq \left(n + \frac{1}{2}\right)\pi$$

e. Sum-to-product identities

$$\sin A \pm \sin B \equiv 2\sin\frac{1}{2}(A \pm B)\cos\frac{1}{2}(A \mp B)$$
$$\cos A + \cos B \equiv 2\cos\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B)$$
$$\cos A - \cos B \equiv -2\cos\frac{1}{2}(A + B)\sin\frac{1}{2}(A - B)$$

f. R-formula (Linear combination of Sine and Cosine)

$$a\sin x \pm b\cos x \equiv R\sin(x \pm \alpha)$$
$$a\cos x \pm b\sin x \equiv R\cos(x \mp \alpha)$$
$$where \; R = \sqrt{a^2 + b^2}, \quad R\cos\alpha = a, \quad R\sin\alpha = b \quad \left(R > 0, 0 < \alpha < \frac{\pi}{2}\right)$$

4. Differentiation

a. Basic functions

$$\frac{d}{dx}(x)^n = nx^{n-1} \ (n \in \mathbb{R})$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \ (x > 0)$$

b. Trigonometric functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -cosec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

c. Hyperbolic functions

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

d. Inverse Hyperbolic functions

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1 + x^2}}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}, \qquad x > 1$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1 - x^2}, \qquad |x| < 1$$

e. Rules of differentiation

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$

Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

5. Integration

a. Basic Integrals

$$\int f(x)dx = F(x) + C, \qquad (where\ F'(x) = f(x))$$

$$\sinh \int x\ dx = \cos\text{h}\, x + C$$

$$\cosh \int x\ dx = \sin\text{h}\, x + C$$

$$\int tanh\ x\, dx = ln\ (cosh\ x) + C$$

b. Rational forms

$$\int \frac{1}{x^2 + a^2}\ dx = \frac{1}{a} tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x^2 - a^2}\ dx = \frac{1}{2a} ln\left(\frac{x - a}{x + a}\right) + C, \qquad (x > a)$$

$$\int \frac{1}{a^2 - x^2}\ dx = \frac{1}{2a} ln\left(\frac{a + x}{a - x}\right) + C\ , \qquad (|x| < a)$$

c. Root forms

$$\int \frac{1}{\sqrt{x^2 + a^2}}\ dx = \sinh^{-1}\left(\frac{x}{a}\right) + C = \ln(x + \sqrt{x^2 + a^2}) + C, \qquad (x > a)$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\ dx = \cosh^{-1}\left(\frac{x}{a}\right) + C = \ln(x + \sqrt{x^2 - a^2}) + C, \qquad (x > a) \qquad (x > a)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\ dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \qquad (|x| < a)$$

d. Numerical Integration (Trapezium Rule)

$$\int_a^b y\, dx \approx \frac{1}{2} h[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$$

$$where, \qquad h = \frac{b - a}{n}, \qquad y_i = f(a + ih)$$

6. Mechanics

a. Kinematics

Average velocity: $v_{avg} = \frac{\Delta s}{\Delta t}$

Average acceleration: $a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of uniformly accelerated motion:

$$s = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

b. Dynamics

Newton's second law: $F = ma = \frac{d(mv)}{dt}$

Impulse: $F \times \Delta t = \Delta(mv)$

Friction: $F \leq \mu R$

c. Work, Energy and Power

Work: $W = Fdcos\theta$

Power: $P = \frac{W}{t} = Fv$

d. Moments

Moment = $Fdsin\theta$

e. Circular motion

Centripetal acceleration: $a = \frac{v^2}{r} = r\omega^2$

Centripetal force: $F = \frac{mv^2}{r} = mr\omega^2$

f. Centre of mass of uniform of bodies

Triangular lamina: $\frac{2}{3}$ $along\ median\ from\ vertex$

Solid hemisphere of radius r: $\frac{3}{8}r$ $from\ centre$

Hemispherical shell of radius r: $\frac{1}{2}r$ $from\ centre$

Circular arc of radius r, angle 2α: $\frac{r\sin\alpha}{\alpha}$ $from\ centre$

Circular sector of radius r, angle 2α: $\frac{2r\sin\alpha}{3\alpha}$ $from\ centre$

Solid cone/pyramid of height$h$: $\frac{3}{4}h$ $above\ the\ base\ on\ the\ line\ from\ centre\ of\ base\ to\ vertex$

Conical shell of height h: $\frac{1}{3}h$ $above\ above\ the\ base\ on\ the\ line\ from\ centre\ of\ base\ to\ vertex$

7. Probability and Statistics

a. Mean of two sets

$$\bar{x} = \frac{n_a\bar{x}_a + n_b\bar{x}_b}{n_a + n_b}$$

where n_i is the size of the set i, and $\bar{x}_i$ is the mean of set i.

b. Mean from a frequency table

$$\bar{x} = \frac{\sum xf}{\sum f}$$

Where f is the frequency of value x.

c. Variance and Standard Deviation

Variance:

$$\sigma^2 = \frac{S_{xx}}{n}$$

Standard Deviation:

$$\sigma = \sqrt{\frac{S_{xx}}{n}}$$

d. Summary Statistics

$$S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n}$$

e. Least Squares Linear Regression

Line of y on x:

$$y = ax + b$$

Where,

$$a = \frac{S_{xy}}{S_{xx}}, \qquad b = \bar{y} - a\bar{x}$$

f. Product Moment Correlation

$$\boldsymbol{r} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

g. Coding (if $y = \frac{x-a}{b}$):

$$b\bar{y} + a = \bar{x}, \quad \sigma_x = b\sigma_y$$

h. Expected value of a Discrete Random Variable

$$E(X) = \sum xP(X = x)$$

i. Variance of a Discrete Random Variable

$$Var(X) = E\left[\left(X - E(X)\right)^2\right] = E(X^2) - \left(E(X)\right)^2$$

8. Normal Distribution $N(\mu, \sigma^2)$

The probability density function of a normal distribution with mean μ and variance $\sigma^2$is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

If Z is a standard normal distribution variable ($\mu = 0, \sigma^2 = 1$), then the cumulative distribution function is:

$$\Phi(z) = P(Z < z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{t^2}{2}} dt$$

For **negative values** of $\boldsymbol{z}$, we use:

$$\Phi(-z) = 1 - \Phi(z)$$

The table on the next page gives $\Phi(z)$ for $z \geq 0$.
To use: take the row for the first two decimals of z, then add the column header for the third decimal place.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	11	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	8	11	15	19	23	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	5	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	5	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	19	22	25	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	15	19	22	23
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	5	8	10	13	15	18	20	23	25
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	9	12	14	16	19	21	24
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	9	12	14	15	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	5	9	10	13	15	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	1	3	5	6	7	9	10	11	13
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	2	4	5	6	7	8	9	11
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	0	2	3	4	5	6	7	8	9
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	0	1	2	3	4	5	6	7	8
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	0	1	1	2	3	4	4	5	6
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	0	1	1	2	2	3	3	4	4
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	0	0	1	1	2	2	3	3	4
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	0	1	1	1	2	2	3	3
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	0	0	1	1	1	2	2	2
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	0	0	1	1	1	1	2	2
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	0	0	0	1	1	1	1	1
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	0	0	1	1	1	1	1
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	0	0	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	0	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	0	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	1

If $Z \sim N(0,1)$, then the z-values corresponding to common cumulative probabilities $P(Z < z) = p$ are:

P	**0.75**	**0.90**	**0.95**	**0.975**	**0.99**	**0.995**	**0.9975**	**0.999**	**0.9995**
Z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291